Probabilistic Design of Sheet-Metal Die by Finite Element Method

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Abstract

In sheet metal forming dies, highest stresses occur in bars which connect die to the frame. Considering fatigue behavior of bars with finite element method and probabilistic analyze provide many advantages predicting the life of dies in the design process. An effective and efficient design strategy is proposed to design sheet-metal die in order to reduce stress and increase fatigue life of sheet-metal die. In this strategy, Finite Element Analysis, Approximate model, a numerical optimization algorithm and probabilistic design method Monte Carlo Simulation are integrated to create an automated design tool. The reliability of the results are checked and refined by using probabilistic design techniques. Finally with the help of probabilistic design results, fatigue behaviors of the bars are predicted. At the end of the analysis process % 43 volumes is gained. Stress value is %46.3 lower than the initial design. Safety factor of fatigue is reduced %36.

Keywords: Reliability, finite element analysis, optimization, fatigue

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1. Introduction

The application of computer-aided engineering (CAE), computer-aided design (CAD), and computer-aided manufacturing (CAM) technologies are essential in sheet metal forming industry. Especially the sheet metal forming dies those are used in the automotive industry works under high loads and therefore have very high costs. Thus, increasing the usage time of the die hard tool is very important for decreasing the total cost in the mass production. In sheet metal forming process, very high stresses may occur in the die structure. Hundreds thousands of times die press the metals. One of the most important factors in the die structure design is to predict fatigue life of hard tool. [1] For this reason, the process of sheet metal die modeling for the investigation and understanding of deformation mechanics and predicted fatigue life has become a major concern in recent and the finite element method (FEM) has gained increasing importance, particularly in the simulation deformation processes and fatigue. Finite element method studies with perfect models but in real life nothing is perfect. Unpredictable design parameters occur in manufacturing processes. To avoid the risks caused by these unpredictable parameters, designers use safety factors. In recent years instead of safety factor, some designers began to use probabilistic design techniques. Probabilistic design techniques can be used spread and effectively even in hard engineering areas such as the behavior of solid and hollow dies in hot extrusion [2], fatigue behavior of welded steel structures [3], and the design of tunnel supports [4].

In sheet metal forming dies, highest stresses occur in bars which connect die to the frame. Considering fatigue behavior of bars with finite element method and probabilistic analyze provide many advantages predicting the life of dies in the design process. Friction and blank holding force can create uncertainties in sheet metal process, and the reliability of the system can be analyzed by using Monte Carlo simulations [5]. Also, even if you have test results for fatigue performance, the reliability of the test results should be considered [6].

In this study, an effective and efficient design strategy is proposed to design sheet-metal die in order to reduce stress and increase fatigue life of sheet-metal die. In this strategy, Finite Element Analysis, Approximate model, a numerical optimization algorithm and probabilistic design method Monte Carlo Simulation are integrated to create an automated design tool. Using this approach, in sheet metal forming dies, highest stresses occur in bars which connect die to the frame is formulated in the form of an optimization problem that can be solved easily by a conventional numerical optimization algorithm. Two types of loading conditions that are used on the hard tool in this study are coil spring and gas spring. The reliability of the results are checked and refined by using probabilistic design techniques. Finally with the help of probabilistic design results, fatigue behaviors of the bars are predicted. Solution of the optimization problem leads to the optimum design. Details of design are given in the following sections.

2. Optimization Algorithm, Finite Element Modeling, Probabilistic Design and Fatigue

2.1 Optimization Method

A shape or material design optimization problem can generally be formulated as a constrained minimization problem as following [7]

minimize:

$$y_0(\mathbf{x}) \tag{1}$$

subjected to:

$$y_j(\mathbf{x}) \le 0 \qquad (j = 1,\dots,n_c) \tag{2}$$

within the design space:

$$x_{il} \le x_i \le x_{iu} \qquad (i = 1,\dots,N) \tag{3}$$

where $y_0(\mathbf{x})$ is the objective function, $y_j(\mathbf{x})$ $(j = 1,\dots,n_c)$ are the constraint functions and $\mathbf{x} = [x_1, x_2, \dots, x_N]$ is the vector of design variables. x_{il} and x_{iu} describe physical upper and lower bounds on design variables. n_c and N are the number of constraints and number of design variables, respectively. The constraint and objective functions may correspond to weight, penetration depth, energy absorption, etc.

Solution of Equations (1)-(3) for shape optimization problems can be efficiently done by replacing objective and constraint functions with their Response Surface (RS) approximations. Optimization with approximations is often referred to as approximate optimization in the literature. The approximate optimization method implemented in ANSYS DO module and used in this study is shown in Fig. 1. ANSYS DO module generates and utilizes polynomial RS approximation for objective or constraint function as following [8]

$$\tilde{y}(\mathbf{x}) = \underbrace{\underbrace{\underbrace{a_0 + \sum_{n=1}^{N} a_n x_n}_{linear} + \sum_{n=1}^{N} b_n x_n^2}_{quadratic} + \sum_{m=1}^{N-1}\sum_{n=m+1}^{N} c_{mn} x_m x_n}_{quadratic+crossterms} \tag{4}$$

where *a*, *b*, *c* are coefficients to be determined.

In design optimization process, ANSYS DO first creates *N*+2 design sets to construct a linear approximation. Here set indicates values of all parameters for a specific design. ANSYS DO will either generate design sets randomly or use the existing ones in the optimization database. Shape optimization analysis is carried out at available design sets. Analysis results are then used to create linear approximations of objective and constraints. Higher order approximations such as quadratic and quadratic with cross terms RS approximations are created using least square method when there are enough design sets in the database. The optimum design is predicted by solving Equations (1)-(3) with a numerical optimization algorithm based on penalty functions. The predicted optimum is verified by exact analysis (ANSYS). If the predicted objective and constraints are identical with the results from ANSYS, or the estimated optimum design is satisfactory enough, the optimization loop is stopped. Otherwise, the newly calculated results are added to the existing design sets and new approximations are created followed by the solution of the optimization problem.

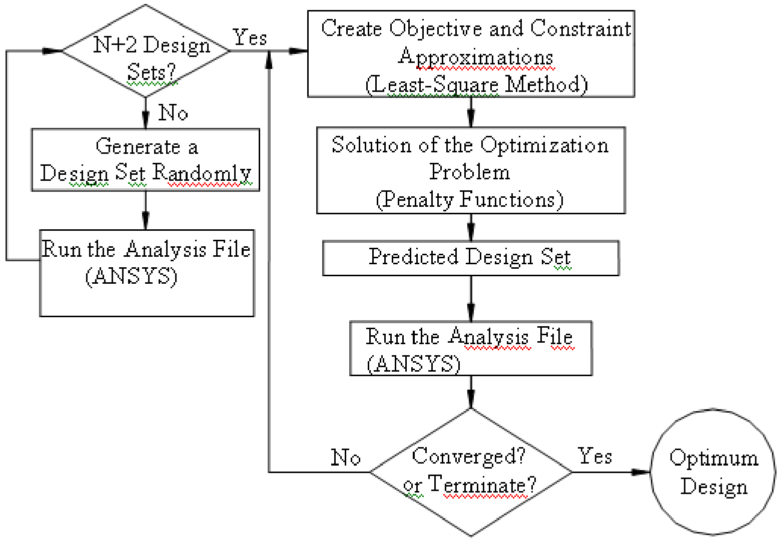
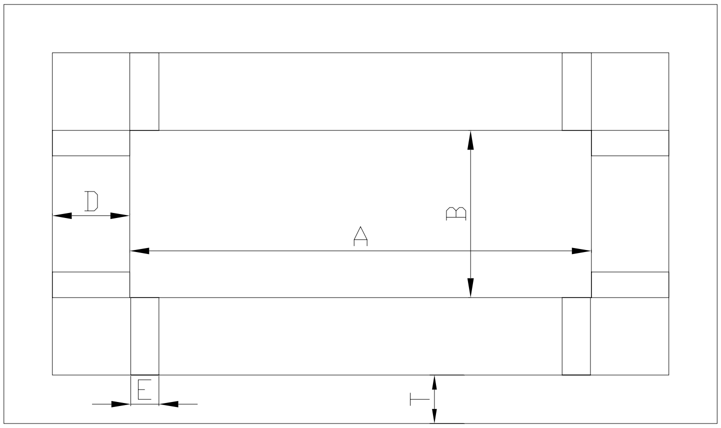


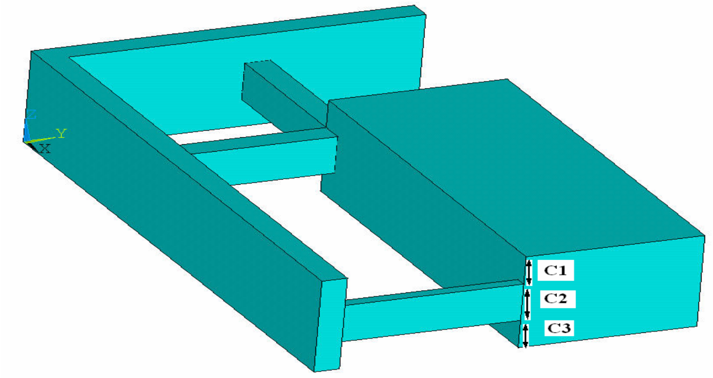
Fig. 1. Approximate design optimization process with ANSYS DO module.

2.2 .Geometric and Finite Element Modeling

Geometry is modeled parametrically as shown in Figure 2 and 3 and obtained from Ford Turkey. High-Roof panel die is used as model. Upper and lower limits of the dimensions are also obtained from Ford.



(a)

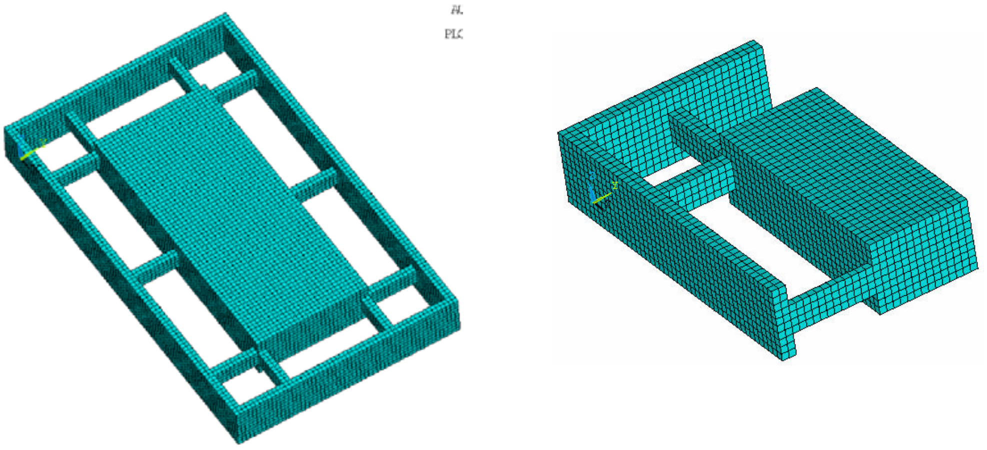


(b)

Fig. 2.Design parameters (a) two dimensional (b) three dimensional

A = 1400 for symmetric model
B = 650 for symmetric model
C1 = 75
C2= 75
C3= 75
D = 200
E = 40 (width of bars)
T = 40 (width of frame)

The first step in the finite element method is to build finite element model equivalent to geometric model. In this work quarter model of the die is meshed with eight nodes solid brick elements. The quarter model is modeled by using 15264 elements. Die were modeled with SOLID45 which is one of the element types in the library of ANSYS. This element type is defined by eight nodes each having three degrees of freedom with linear shape functions. It is shown in Figure 3.



(a) (b)

Fig. 3 Finite element modeling (a) fully model (b) symmetric model

The second step of finite element method is to select material models which represent behavior of materials. Because of the stress and strain values on the die occur in the elastic limit, linear homogeneous elastic material type is selected.
GG30 Ductile Casting Material:
Young Modulus (E): 210000 MPa
Poisson Ratio (υ): 0.3
Density: 7.86e-6 kg/mm^3

The third step of finite element method is to apply loading and boundary condition. Quarter model with symmetry boundary conditions is used. Dynamic load conditions are applied to the model. Also static body weight load is considered. Coil spring and gas spring load conditions are applied respectively as shown in Figure 4. The other aim of this study is to predict the fatigue life of the system according to loading conditions, coil spring and gas spring.

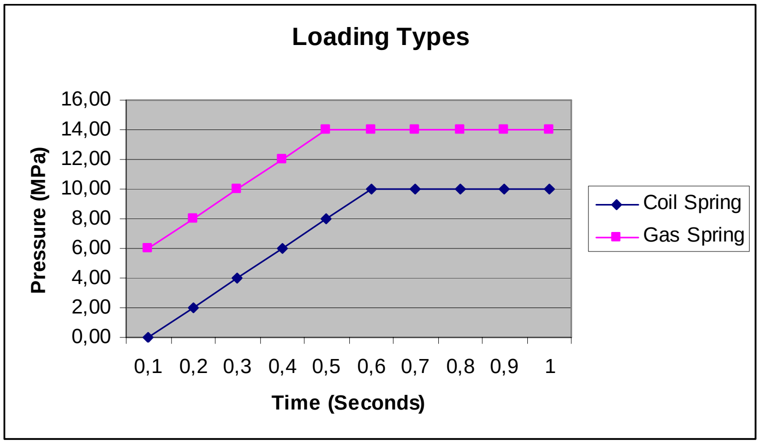
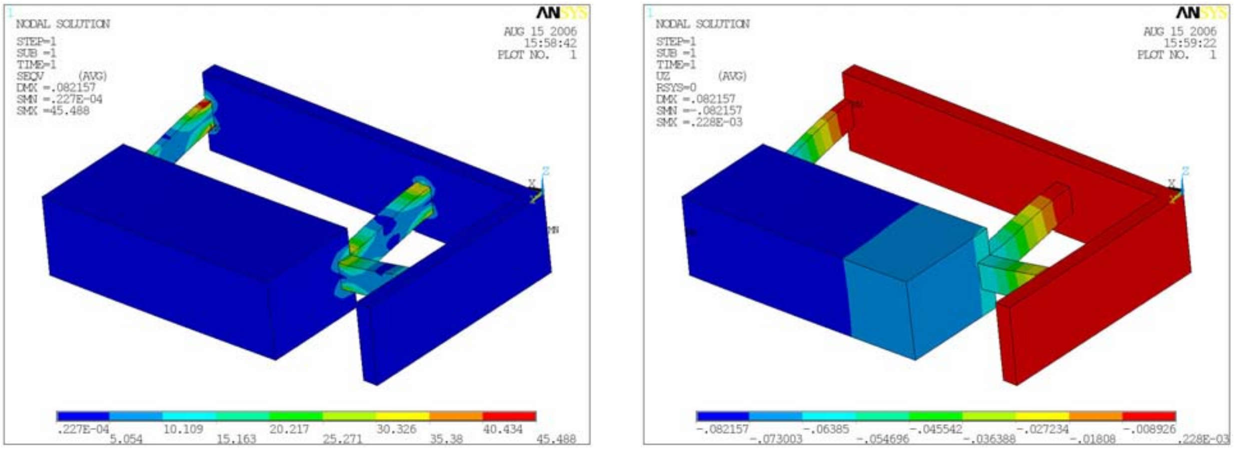


Fig. 4 Loading type characteristics of the model.

3. Results

3.1 Finite Element and Optimization Results

Results of the finite element solution are acceptable. Maximum stress occurred on the root of the bars as expected. Maximum deflection occurred on the center as shown in Figure 5. Finite Element Analysis, Approximate model, a numerical optimization algorithm are integrated to create an automated design tool. 43% reduction on volume is gained . Results of optimization are given in Table 1. Probabilistic approach is used to get better results after optimization process. Details of probabilistic approach are given in the following sections.



(a) (b)

Fig. 5 Finite Element Solution of the model: a) Nodal Solution of Von Mises Stress
b) Nodal Solution of Displacement

Table 1. Optimization Results of the model.

	Initial Design	Range	Optimized Design	Reduction %	Catalog Values
A (mm)	1400	1000-1500	1112,000	20,58	1100
B	650	500-700	529,000	18,6	530
C1	75	30-90	34,824	53,6	35
C2	75	30-90	44,601	40,6	45
C3	75	30-90	34,592	54	35
D	200	180-210	184,570	7,8	185
E	40	24-48	33,000	17,5	35
T	40	24-48	28,000	30	30
Max.Stress (MPA)	85,000		45,500	46,5	
Volume (mm^3)	92000000		52300000	43,2	

3.2 Probabilistic Design

In serial manufacturing processes, every product differs from the others slightly. These differences are the cause of uncertainties in material, human factor, dimensioning, machine settings, etc. If there are enough samples, each design parameters can be graphed and frequency of each design parameters leads to probability of that design parameter. After that moment each design parameter called as random variable. In traditional engineering calculations mean value of each random variable is used. However random variables are not constant and change in a range. If the ranges of the results are important for the design, some other values should be used more than mean value such as standard deviation. In standard calculations, the range factors in the results are eliminated by using safety factors. In probabilistic design, probability distributions of the design parameters are calculated. Probability distribution results show the reliability change of the design. Designer uses the reliability values according to customer desires. This process leads to maximum safety and quality with minimum cost.

The distributions of design parameters or independent random variables show a type. Most common used distribution types are normal, lognormal, uniform, exponential, triangular, beta, gamma, and weibull. The distribution types of the random variables are as shown below.

Table 2. Distribution types of variables.

Random Variable	Distribution Type	Standart Deviation (% of mean value)
A	Lognormal	10
B	Lognormal	10
C1	Lognormal	10
C2	Lognormal	10
C3	Lognormal	10
D	Lognormal	10
E	Lognormal	10
T	Lognormal	10
Young Modulus	Gauss	10
Pressure	Lognormal	10
Density	Uniform	± 5

3.3 Monte Carlo Simulation

Flow chart of Monte Carlo Method which is shown in Figure.6 is one of the techniques that edit the values of the parameters at each loop according to their probability of occurring. In Monte Carlo Method, uniform distributions of random numbers are generated. These values are transformed to numbers by using one of the parameter's cumulative distribution functions. Number transforming process is done according to chosen parameter's probability density function. To reach a similar distribution type, large numbers of samples are required.

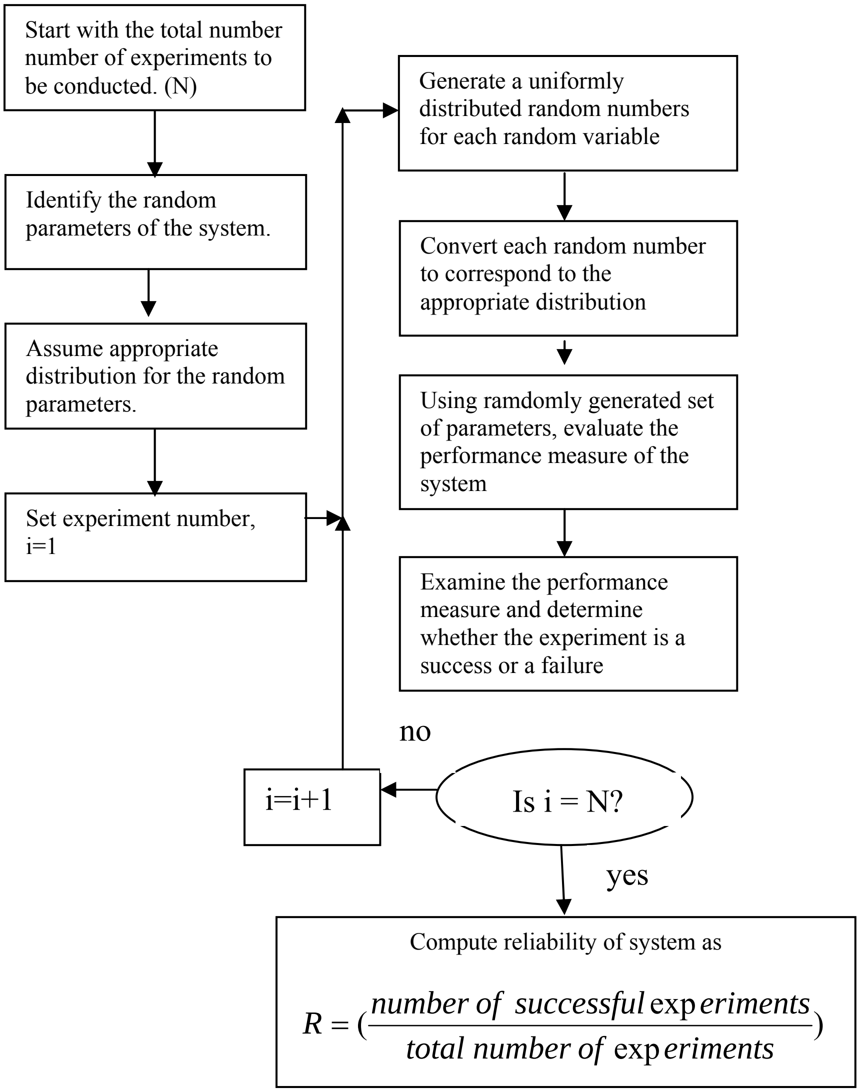


Fig. 6. Algorithm of Monte Carlo Simulation

3.4 Results of Probabilistic Design

By using probabilistic design techniques, stress-reliability diagram is obtained. As seen from the diagram stress increases with reliability. For 50% reliability, the stress value is about 43 MPa for spring coil loading conditions. It means that stress will be less than 43 MPa with the probability of 50%. For 99% reliability, stress values are about 80 and 100 MPa for spring coil and gas spring loading conditions respectively. If the system is designed to resist 80 MPa stress for spring coil loading conditions, it will be durable with the probability of 99% as shown in Figure 7.

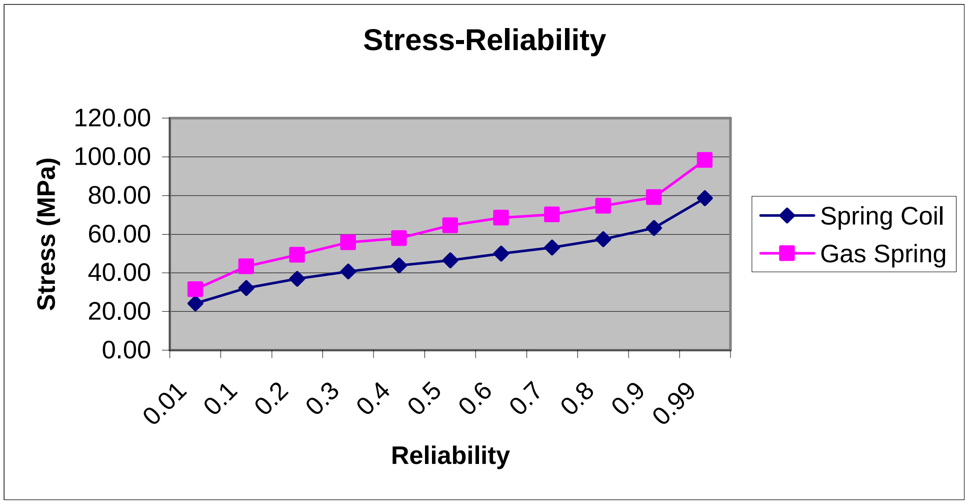


Fig. 7. Stress-Reliability Diagram of loading types.

Safety factor is also obtained more effectively by using probabilistic design techniques. As seen in diagram below, safety factor of 1.8 is enough for 99% reliability. Generally 2.5-3 safety factor coefficients are used in die design. Here by using probabilistic design techniques, it is seen that 1.8 safety factor is enough as shown in Figure 8. That causes about 30-40% less cost and weight profit. For gas spring loading condition, safety factor is about 1.5 and the profit is much more.

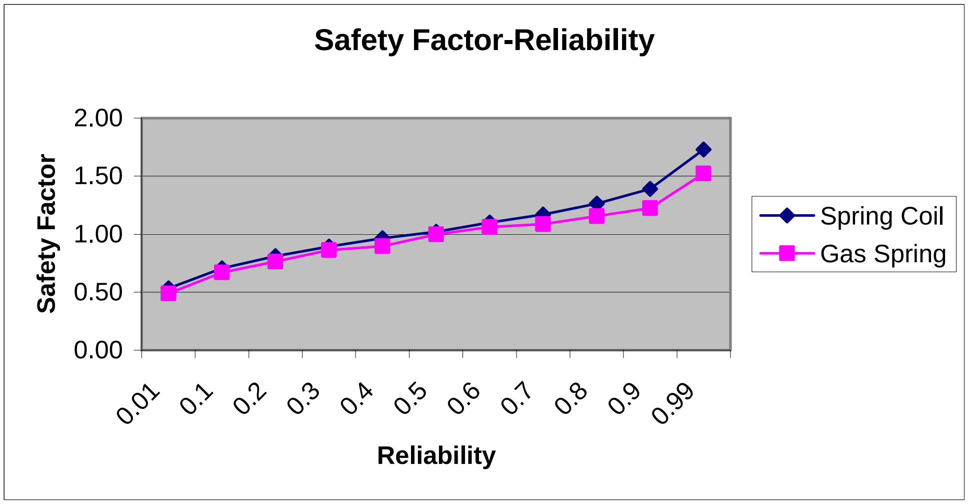


Fig. 8. Safety Factor-Reliability Diagram of loading types.

Sensitivity of the random variables on stress and deflection is also researched and is shown in Figure 9. It is obtained that C2 is the most significant variable on stress. A, Pressure, B, D, E, and C3 are the other significant variables. C2, E and C3 are opposing variables on stress. Higher values of these mean less stress. Young Modulus and Density are insignificant on stress as expected. T and C1 are also insignificant on stress.

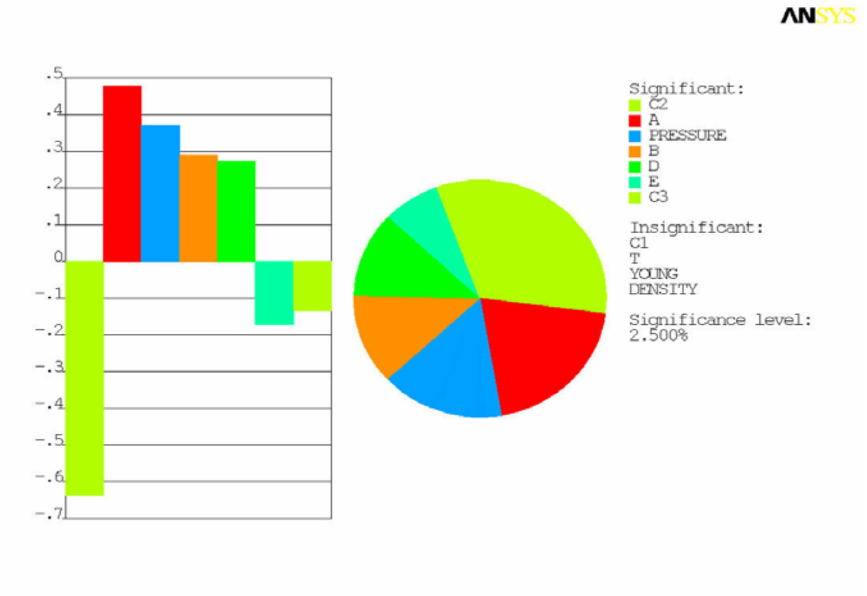


Fig. 9. Sensitivity of the random variables on stress

C2 is the most significant variable also on deflection. D, A, Pressure, Young Modulus and B are the other significant variables on deflection. C2 and Young Modulus have reverse relation on deflection. C1, C3, E, T and Density have no effect on deflection.

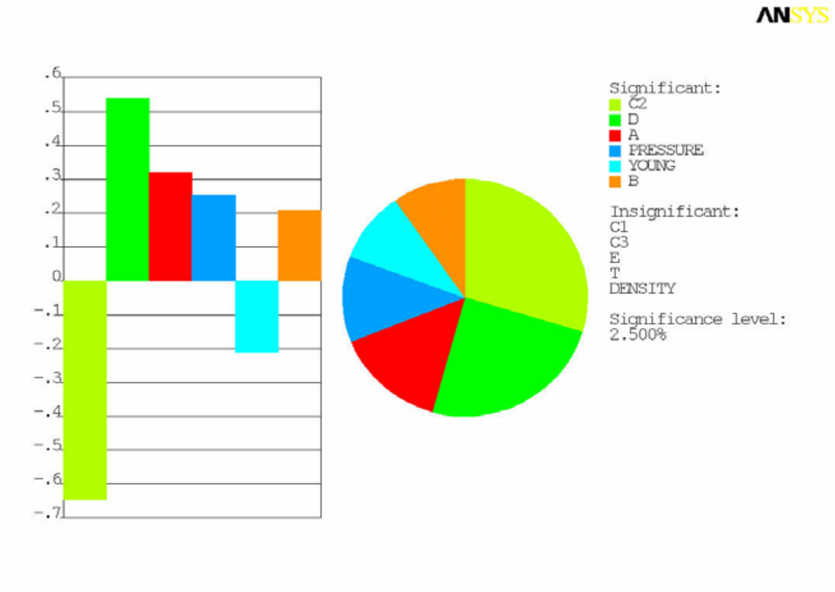


Fig. 10. Sensitivity of the random variables on displacement

The Matrix Correlation coefficients for the random variables are shown below in Table.3. The values in brackets are insignificant variables. C2 values are (-) for both stress and deflection because of opposing relation.

Table 3. The Matrix Correlation coefficients for the random variables

	A	B	C1	C2	C3	D	E	T
Max. Deflection	0.319	0.209	(0.048)	-0.647	(-0.087)	0.538	(-0.126)	(-0.018)
Max Stress	0.478	0.291	(0.052)	-0.638	-0.133	0.275	-0.172	(-0.052)

	Young	Pressure	Density
Max. Deflection	-0.211	0.254	(0.035)
Max Stress	(0.010)	0.373	(-0.007)

3.5. Fatigue of Die

Under cyclic loading, strength of materials reduces by time and fracture can occur at lower stresses than ultimate stress. This happens because of fatigue. In die design, fatigue calculations also take place. Dies are used hundreds thousands of times in automobile industry and bad designed dies fail because of fatigue. In the sheet metal processes, wrinkles are related with many parameters; press force, blank holder force, and timing, draw bit design etc. Wrinkles cause stress concentrations and so cracks as shown in Figure 11. With cyclic loading cracks propagate and die fails. The reason for a sheet-metal die design is to obtain reliable and have a long fatigue life.

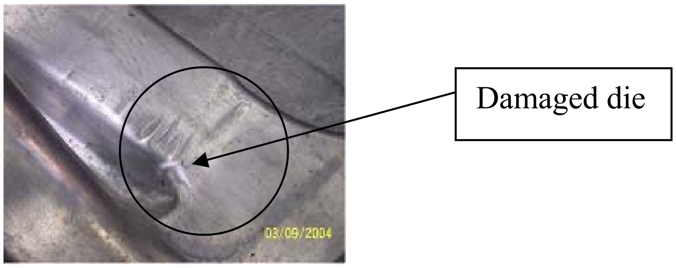


Figure 11. Damaged Die in sheet metal forming process

Also in hard tools, maximum stress occurs mostly on bars. Well-designed bars endure against cyclic loading and fatigue. Maximum endurance with minimum cost is the aim. Here probabilistic design takes an important place.

In this study, fatigue life of the die upon finite element stress analysis is predicted using the computer code of ANSYS/Workbench [8]. Fatigue life of die is calculated based on Goodman, Soderberg, and Gerber fatigue theories which are illustrated in Table 4.

Table 4 Fatigue theories and formulas used in fatigue life predictions.

Fatigue Theories	Fatigue Formulas
Goodman	$(\frac{\sigma_a}{S_e}) + (\frac{\sigma_m}{S_u}) = \frac{1}{N}$
Soderberg	$(\frac{\sigma_a}{S_e}) + (\frac{\sigma_m}{S_y}) = \frac{1}{N}$
Gerber	$(\frac{N\sigma_a}{S_e}) + (\frac{N\sigma_m}{S_u})^2 = 1$

In Table 4, N indicates safety factor for fatigue life in loading cycle, S_e for endurance limit and S_u for ultimate tensile strength of the material. Mean stress σ_m and alternating stress σ_a are defined respectively as

$$\sigma_m = \frac{(\sigma_{max} + \sigma_{min})}{2} \qquad (5)$$

$$\sigma_a = \frac{(\sigma_{max} - \sigma_{min})}{2} \qquad (6)$$

Von Misses stresses obtained from finite element analyses are utilized in fatigue life calculations.

All fatigue analyses are performed according to infinite life criteria (i.e. $N=10^9$ cycles)

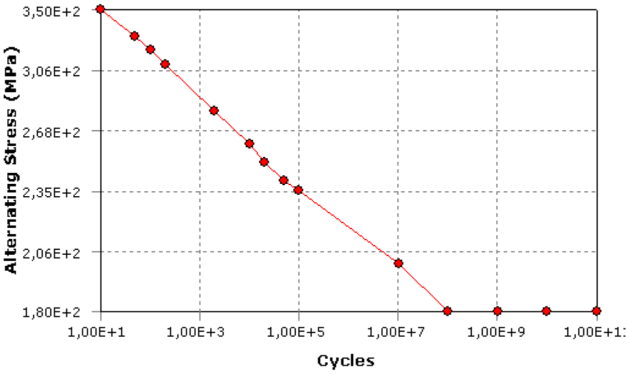


Figure 12. S-N curve of GG-25

3.6 Reliability for Fatigue

According to fatigue theories explained above, a safety factor is necessary for infinite cycle of loading. If fatigue safety factors are calculated by using Goodman, Soderberg and Gerber, safety factor coefficients are found respectively, 5.65, 5.23, and 5.72. These mean that if the part is designed for 5.23 safety factor, it will be used for infinite times. However, if the reliability of the system is considered, more efficient fatigue safety factor would be gained. It is seen above that for maximum stress calculations C2, A and Pressure random variables are the most significant variables. By using these three variables fatigue reliability is calculated. As shown in figure 13 below, safety factor coefficients are reduced from 5 to 3.2 which mean %36 profits from material, labor, etc.

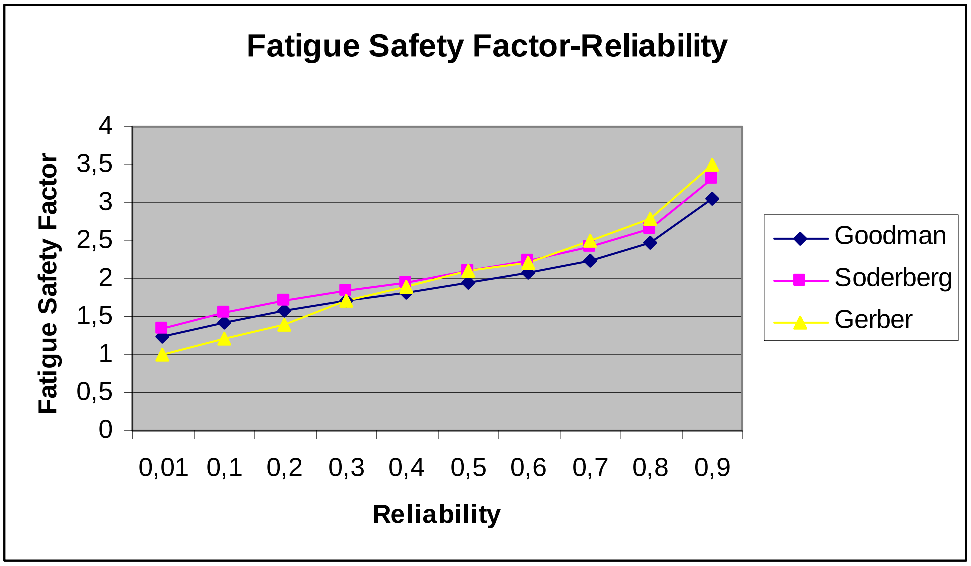


Fig.13 Fatigue Safety Factor-Reliability diagram for fatigue theories

4. Conclusion

- Volume reduction can be handled up to %43 with the same maximum stress values by using design optimization techniques.
- Gas spring loading condition applies about %25-40 much more pressure on the system than coil spring.
- Maximum stress is occurred on the root of the bars and maximum deflection is on the center of the mass.
- Probabilistic design techniques show that 1.8 safety factor will be enough for %99 reliable designs. (Without fatigue calculations). It also show safety factor 1 is about %50 reliable.
- Stress is affected mostly from C2, A, and Pressure values respectively. Stress is dependent from Young Modulus and Density.
- Displacement is affected mostly from C2, D, A, Pressure and Young Modulus values respectively. Displacement is dependent from density. Young Modulus is significant on displacement rather than stress.

- Fatigue calculations give safety factors between 5.2-5.7. However by using probabilistic design techniques it is seen that safety factor 3.2-3.5 is enough for infinite cycle. It is about twice the safety factor value of 1.8 mentioned above. Safety factor 1 will definitely fail under cycling loading.
- Probabilistic design techniques supply to predict more efficient safety factors for static, dynamic, and cycling loading conditions. More efficient safety factor means more light and cheap designs.

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