Multi-response optimisation in a three-link leaf-spring model

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Abstract: The simulation of road loads in truck suspensions generally requires leaf-spring models. There are many models in the literature. In this study the three-link leaf-spring model was compared with the test results. An attempt was made to establish the model by predicting accurate results. In order to perform this task, standard tests were performed to obtain the relationship between the torque and angular displacement. After that the corresponding three-link model was generated, and run using the ADAMS solver. After obtaining the desired model the parameters effecting the acceleration for the driver seat were optimised by means of an ANOVA method. The parameters are the length of the leaf spring, the material properties of the leaf spring and the stiffness of the springs connected to the front panel. The reduced acceleration for the driver seat was determined by the response surface regression method.

Keywords: leaf spring; optimisation; three-link.

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Biographical notes: Bülent Ekici is currently an Associate Professor of Mechanical Engineering at Marmara University. He received his BS, MS and DSc degrees from Bogaziçi University. He is an active product designer who has taught, researched and written about computer aided design, manufacturing and engineering for over ten years. The author has more than 5000 attendees for his CAD and CAM seminars arranged for industry. He also professionally deals with rapid prototyping and silicon moulding technology in industry.

1 Introduction

Analytical simulation of road loads is an enabling technology for CAE-based durability analysis. Significant progress has been made in this area over the last few years. A few years ago, all road loads were directly measured. Today, however, semi-analytical loads make up the majority of loads delivered to vehicle programmes (Işık Erzi, 2001). Semi-analytical loads are calculated using analytical models, which use measurements on prototypes as inputs. Semi-analytical methods have been used on most suspension configurations except that of Hotchkiss. The reason is the lack of validated leaf-spring models under large deflection dynamic conditions in the chassis suspensions.

Vehicle system models built using ADAMS (2002) software currently represent a leaf spring as a 'three-link model', or a 'beam model'. The state-of-the-art is the beam model where, each spring leaf is discretised into a series of rigid parts, connected using massless linear beams of uniform cross sections. Adjacent leaves of the spring are kept from penetrating each other using ADAMS 'IMPACT' statements. This results in a very large, and extremely non-linear model with several hundred degrees-of-freedom. It was found that it is difficult to perform road-load simulations using this type of leaf spring model (Tagawa et al., 1995).

The SAE Spring Design Manual (1982) presents a three-link discretisation of a leaf spring for analysing its kinematics. The manual does not specify how to compute the force-deflection behaviour and employs the SAE three-link model with linear torsional springs to model leaf springs for vehicle handling simulations. It is expected that the linear approximation will not hold in the case of multistage springs under large deflections. An extension to the three-link model uses a non-linear vertical spring in parallel with the three-link model to account for the non-linear force-deflection behaviour in the vertical direction. Although this model is useful in certain applications, the spurious load path introduced through the vertical spring and the likely misrepresentation of longitudinal behaviour make this model unsuitable for applications in road load simulation.

There are other relevant studies reported in literature which include the work by Tagawa et al. (1995). His study presents analytical formulations for leaf spring force-deflection behaviour. The results show good correlation with test data. The formulation does not address multistage springs, where not all the leaves are always in contact. In addition, ADAMS implementation of this and other analytical formulations is not straightforward.

The leaf-spring model presented in this study is a 'three link' model. The geometry of this model consists of three rigid links constructed using the SAE guidelines (1982). The leaf-spring compliance is incorporated in the model through two non-linear torsional springs at the centre-link joints. A systematic approach is presented for computing the parameters of the non-linear springs (Jolly, 1983).

The automotive industry is working on various types of suspension systems because as time goes by expectations of customers about vehicle comfort increases. For example, very serious works are performed on electro-pneumatic suspensions, intelligent suspensions, active and semi-active suspension control systems (Patent, 2002). Nowadays some firms in Europe are working with a complicated test. This system helps the engineers obtain a large range of data.

Although passive suspension systems may be replaced with active suspension systems in the future, leaf-spring suspension systems are still in use today. There are many factors affecting the performance of leaf springs. In this study the length of the leaf spring, the ratio of the elastic modulus to yield strength and the stiffness on the front panel of the truck were selected for evaluation (Hales, 1985). The effects of these factors on the acceleration of the chassis, on the front panel and under the driver's seat are investigated due to the road input. By means of the data collected from experiments performed in an Instron test machine a model is created in the ADAMS solver. The results obtained from this model are used in this optimisation study.

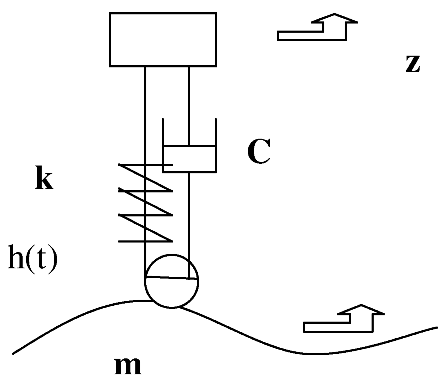
2 Materials and methods

2.1 Mathematical models for a quarter vehicle model

In order to establish the suspension model, the de Carbon model has been taken as a base for forming the quarter vehicle model.

In the de Carbon model, as shown in Figure 1, the wheel of the vehicle is assumed to be elastic without damping in order to find a linear solution. But for our model for heavy truck suspension it was not possible to obtain the elasticity value. Also, the real road-load data was collected from the axle and chassis.

Figure 1 De Carbon model



For these reasons, in our model the wheel was assumed as rigid, and assumed to be transmitting road irregularities directly to the axle (undamped mass). On our model system, because of limitations of the software; the rubber bushings' friction surfaces are not simulated. According to these restrictions, the equation of motion is:

$$m\ddot{z} = -c(\dot{z} - \dot{h}(t)) - k(z - h(t)). \tag{1}$$

If the equation is reorganised

$$m\ddot{z} + c\dot{z} + kz = c\dot{h}(t) + kh(t). \tag{2}$$

Assuming a harmonic excitation in the form

$$h(t) = b.\sin(\omega.t), \tag{3}$$

then a special solution of the equation of motion is:

$$z(t) = A\sin(\omega.t) + B\cos(\omega.t). \tag{4}$$

The amplitude of the motion can be solved as:

$$a = \sqrt{A^2 + B^2}. \tag{5}$$

Acceleration amplitude for vibration analysis is:

$$|\ddot{z}| = w^2a^2. \tag{6}$$

In order to apply these motion equations correctly to our model; the sprung and unsprung bodies must be rigid on the vehicle that the real road-load data has been collected from, the road-load must be single harmonic data not a sum of different harmonics (a test road with this kind of profile is not available in our country).

2.2 Testing

The leaf spring tests were performed by using an Instron hydraulic testing machine and road data were collected with LMS Road-Runner data acquisition software and computer. Although for road runner data, the test was performed for a 85,000 km road, only a 35-min portion was selected. This portion was selected from different parts of the 85,000 km. The reason for that is to select the worst conditions that the truck will confront.

In the study, leaf springs of a load-carrying vehicle were studied. The leaf springs are fixed to the floor from the centre line of the axle. Load is applied to one end of the leaf spring while the other end is fixed. Load applied on the spindle of the Instron is measured for every 6 mm deflection of the end of the leaf spring, then the same procedure is applied to the other end of the spring.

The data obtained from the test is used to model the springs in ADAMS by applying the 'Three-Link Leaf-Spring Model for Road Loads'. The material of the leaf springs studied is 55Cr3 steel. The displacement of the load-applying spindle is used to calculate the angular deflection of the torsional spring attached to the links in the model.

From the known values of the applied load and the deflection of the spring (Table 1), values of torsional load and angular deflection of the proposed model are calculated (Table 2). The relation between the torsional stiffness and the angular deflection is derived as a third order polynomial by using Matlab (2002).

By using the data obtained from tests (Table 1), the equivalent of a three leaf-spring model is found. The dimensions of the equivalent SAE linkage mechanism for the leaf springs for the front axle are explained in Table 1.

Using these dimensions and the displacement of the Instron's piston, the displacement angles are calculated. The torsional load on the torsional springs attached to the ends of the base linkages in the model are calculated by multiplying the perpendicular component of the load applied by the piston with the SAE equivalent dimensions of the linkages. The displacement of the load-applying spindle is used to calculate the angular deflection of the torsional spring attached to the links in the model. From the known values of applied load and the deflection of the spring, values of torsional load and angular deflection of the proposed model are calculated. The relation between the torsional stiffness and the angular deflection is derived as a third-order polynomial by using Matlab. Torsional loads versus degrees of displacement of the modelled links for each side of the leaf springs are given in Table 2.

Table 1 Front axle leaf spring-load versus displacement

	1st side (573.75 mm)			*2nd side (573.75 mm)*		
Displacement (mm)	*Load (up) (kN)*	*Load (down) (kN)*	*Load (ave) (kN)*	*Load (up) (kN)*	*Load (down) (kN)*	*Load (ave) (kN)*
0	0	−0.2	−0.1	0	−0.7	−0.35
6	0.7	0.1	0.4	0.7	−0.3	0.2
12	1.1	0.8	0.95	1.5	0.2	0.85
18	1.7	0.9	1.3	2.2	0.8	1.5
24	2.1	1.3	1.7	2.8	1.3	2.05
30	2.5	1.7	2.1	3.5	1.9	2.7
36	3	2.1	2.55	4.2	2.6	3.4
42	3.5	2.6	3.05	4.9	3.1	4
48	4	3	3.5	5.3	3.7	4.5
54	4.6	3.5	4.05	6.2	4.3	5.25
60	5.2	4.1	4.65	6.6	5.3	5.95
66	5.8	4.9	5.35	7.7	5.7	6.7
72	6.5	5.6	6.05	8.3	6.4	7.35
78	7.2	6.2	6.7	9	7	8
84	8	6.9	7.45	9.6	7.7	8.65
90	8.6	7.9	8.25	10.2	8.3	9.25
96	9.3	8.2	8.75	10.9	9	9.95
102	10	8.9	9.45	11.5	9.7	10.6
108	10.8	9.6	10.2	12.2	10.4	11.3
114	11.5	10.7	11.1	12.8	11.1	11.95
120	12.2	11.1	11.65	13.5	11.8	12.65
126	13	11.8	12.4	14.1	12.5	13.3
132	13.7	12.6	13.15	14.8	13.3	14.05
138	14.5	13.5	14	15.4	14.1	14.75
144	15.3	15.2	15.25	16.2	15.9	16.05

The relation between torsional loads and degrees of displacement of the modelled links are given in Figure 2(a) and (b).

Polynomial expressions for the leaf spring model for the front axle leaf spring obtained from the data in Table 2 are:

$$\text{1st Side:}\ \ T = -357.4\,x^3 + 25297.5\,x^2 + 297178\,x + 30398.6 \tag{7}$$

$$\text{2nd Side:}\ \ T = 226.4\,x^3 + 1060\,x^2 + 570282.3\,x - 230602.9 \tag{8}$$

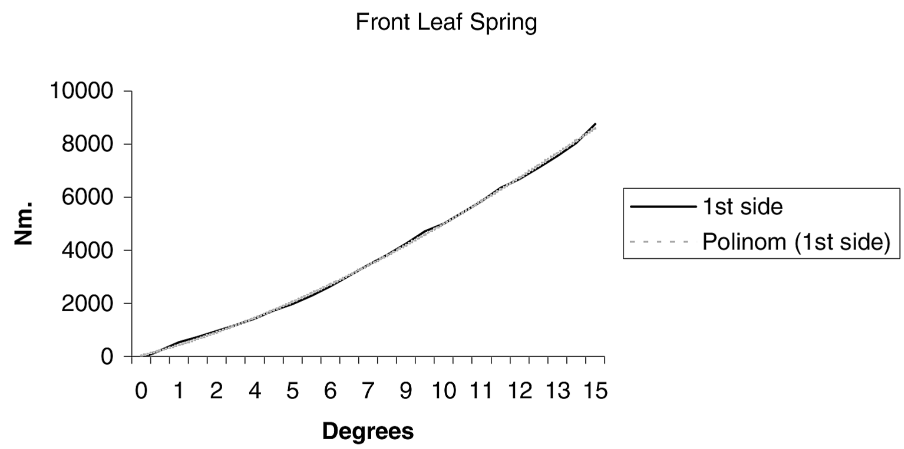
where T is in Nmm and x is in degrees.

Table 2 Front axle leaf spring – torsional loads versus degrees of displacement

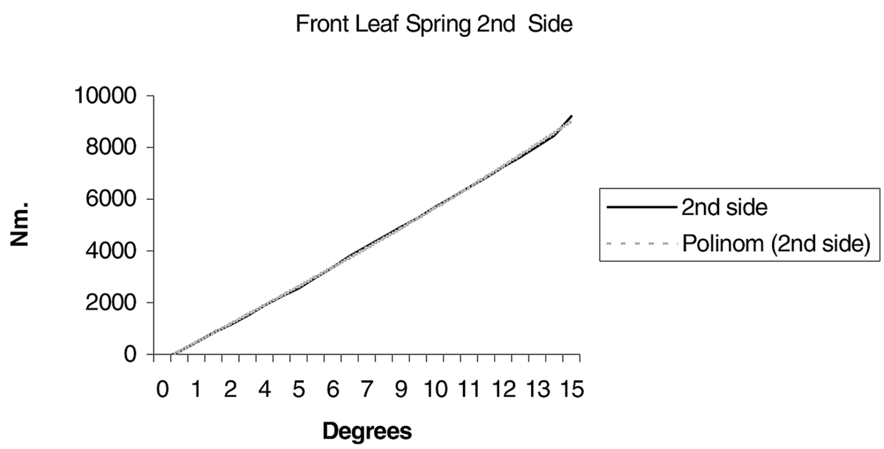
1st side (573.75 mm)		*2nd side (573.75 mm)*	
Displacement (deg)	*T (Nm)*	*Displacement (deg)*	*T (Nm)*
0	−55.539	0	−193.048
0.6181	222.763	0.622	110.65
1.2346	530.441	1.243	471.634
1.8495	727.667	1.861	834.611
2.463	953.803	2.479	1143.66
3.075	1180.85	3.094	1510.08
3.6857	1436.91	3.708	1906.14
4.2952	1722.06	4.32	2247.61
4.9035	1979.82	4.931	2533.98
5.5107	2294.92	5.541	2962.29
6.1169	2639.19	6.15	3363.65
6.7221	3041.06	6.758	3794.38
7.3265	3443.75	7.364	4169.42
7.93	3818.61	7.97	4545.16
8.5328	4251	8.574	4921.49
9.135	4712.43	9.178	5269.78
9.7366	5002.71	9.781	5675.38
10.338	5407.39	10.38	6052.71
10.938	5840.72	10.99	6458.71
11.538	6359.91	11.59	6836.12
12.138	6678.34	12.19	7242
12.738	7111	12.79	7619
13.337	7543.16	13.39	8052.91
13.936	8032.06	13.99	8457.67
14.536	8749.69	14.59	9205.95

Figure 2 (a) Torsional loads and degrees of displacement of the modelled links; (b) torsional loads and degrees of displacement of the modelled links

(a)



(b)



2.3 *Modelling*

After obtaining the relation between torque and angular displacement, the next step is to create a model in the computer. The proposed model consists of three rigid links. The centre link is attached to the end links through torsional springs. The geometry of the model is constructed using the physical spring dimensions and the SAE guidelines (1982) on leaf-spring design. The parameters of the torsional springs are identified from a given design-intent vertical force-deflection curve or from measured force-deflection data. Alternately, the force-deflection curve may be derived from a detailed non-linear finite element model of the spring based on design geometry. The elegance of the proposed method is that a simple model can be easily constructed to reproduce both the kinematics and compliance properties of the actual leaf-spring.

2.3.1 Three-link model geometry

The basis of the SAE three-link geometry is the assumption that a leaf-spring conforms to the shape of a circular arc under vertical loading. Based on The SAE Spring Design Manual (1982), an equivalent five-bar mechanism (four moving bars, including the shackle) can be constructed that closely approximates the kinematics of the leaf spring. To construct the mechanism, only the lengths of the four links are needed. The shackle is represented as it is, with its physical length. The lengths of the other three links are determined as described below.

2.3.2 Dimensions of the three-link mechanism

The dimensions required to build the three-link mechanism are obtained from the geometry of the leaf spring, which is given in Figure 3. Figure 4 shows the key geometric information needed to build the three-link model, where:

L = total spring length, measured along flat main leaf
m = front inactive length
n = rear inactive length
a = fixed cantilever length, called front length (including inactive length, m)
b = shackled cantilever length, called rear length (including inactive length, n).

Figure 3 Layout of a leaf spring

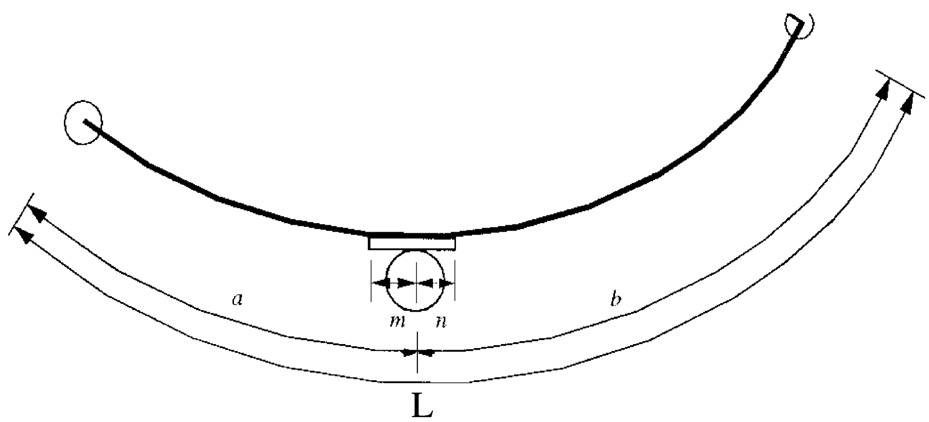
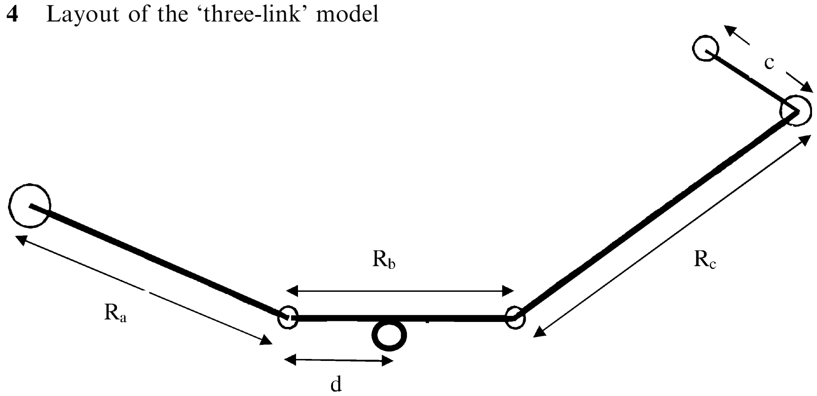


Figure 4 Layout of the 'three-link' model



The 'inactive' lengths, m and n, together make up the centre portion of the leaf spring where the spring is attached to the axle using U-bolts and other hardware. The physical layout is constructed such that the centre portion of the leaf spring is constrained from undergoing any bending. This portion of the leaf spring, which is assumed to be rigid, is called the inactive portion. Based on the SAE guidelines (1982), the dimensions of the equivalent SAE Linkage mechanism are shown in Figure 4 and given as:

$$R_a = 0.75(a - m) \tag{9}$$

$$R_b = 0.75(b - n) \tag{10}$$

$$R_c = L - (R_a + R_b) \tag{11}$$

$$d = a - R_a. \tag{12}$$

2.3.3 Three-link model compliance

To simulate the compliance characteristics of the physical leaf spring, two torsional springs are attached at each end of the centre link. The parameters of these torsional springs are determined such that the vertical static force-deflection behaviour of the physical leaf spring matched the equivalent three-link model. The vertical static force-deflection characteristic of a physical spring is specified during the suspension design process, and hence, is available a priori. The torsional springs at each end of the centre link are represented by the following equations:

$$T_4 = [a_1 \ a_2 \ a_3]\left[\theta_4 \ \theta_4^2 \ \theta_4^3\right]^T \tag{13}$$

$$T_6 = [b_1 \ b_2 \ b_3]\left[\theta_6 \ \theta_6^2 \ \theta_6^3\right]^T \tag{14}$$

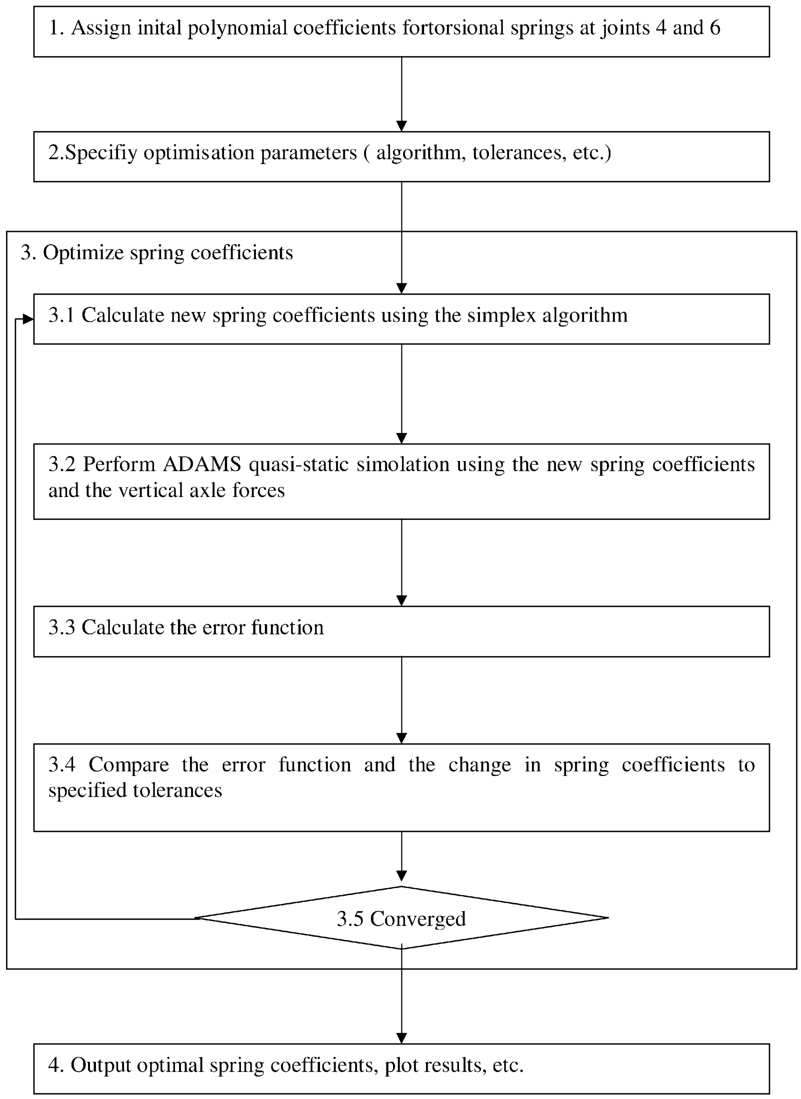
where,

T = torque at the link joint
θ = rotation at the link joint
a_i, b_i = torsional spring parameters, i = 1, 2, 3
4, 6 = node numbers for the centre-link joints.

Equations (7) and (8), where the torques in the link joints are assumed to be third-order polynomials of the corresponding joint rotations, worked well for the leaf spring used in the case study. Typical design curves of multistage leaf springs are specified as piecewise linear, rather than smooth. However, polynomials are chosen because the measured vertical force-deflection behaviour of leaf springs exhibits smooth non-linear characteristics.

The polynomial coefficients, a_i and b_i, i = 1, 2, 3, are identified by 'curve-fitting' the force-deflection behaviour of the physical leaf-spring. Mathematically, this is posed as an optimisation problem where the error between the model and the target curve is to be minimised. The optimisation process is implemented in Matlab, which repeatedly invokes ADAMS to perform the simulation of the linkage mechanism. The procedure is shown schematically in Figure 5.

Figure 5 Optimisation process to identify the torsional spring parameters



The error function in Step 3.3 of Figure 5 is defined as:

$$e_x = ((x_m - x_s)/x_s) \tag{15}$$

$$\text{Error} = e_x^T \cdot e_x \tag{16}$$

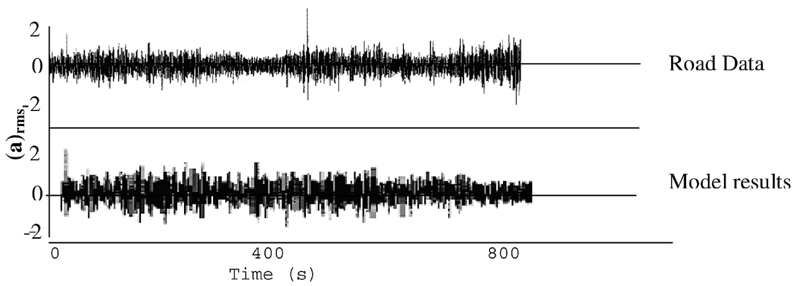
where,

x_s = target values of axle vertical displacements for a series of quasi-static vertical axle force input. The target may be the design specification of actual test data.
x_m = axle vertical displacements of the three-link ADAMS model for the same axle forces.

3 Results of model solution

Real road-load data graphs are collected from Heavy Truck named Cargo 2520 front axle and chassis. Data were collected using Road-Runner data acquisition software. Output graphs of the three-link beam model created with ADAMS software are in accord with the test results. The test values and ADAMS model values were compared by means of a CADA-X programme. A comparison of road data and model results is given in Figure 6.

Figure 6 Comparison of model and test data results in CADA-X



The root mean square values of acceleration for real road data and model results are 1.584 m/s^2 and 1.663 m/s^2 respectively.

4 Optimisation of parameters for reduced acceleration under driver seat

The model created was determined to be suitable for optimisation purposes. The main aim of this study is to obtain a comfortable acceleration response under the driver's seat for real road-loads as well as a reasonable response for the leaf spring. In order to minimise the acceleration for this part of the truck a series of simulations were performed. Figure 7 shows the constraint points studied on the truck. The factors studied are given in Table 3.

Figure 7 The optimised model for driver seat

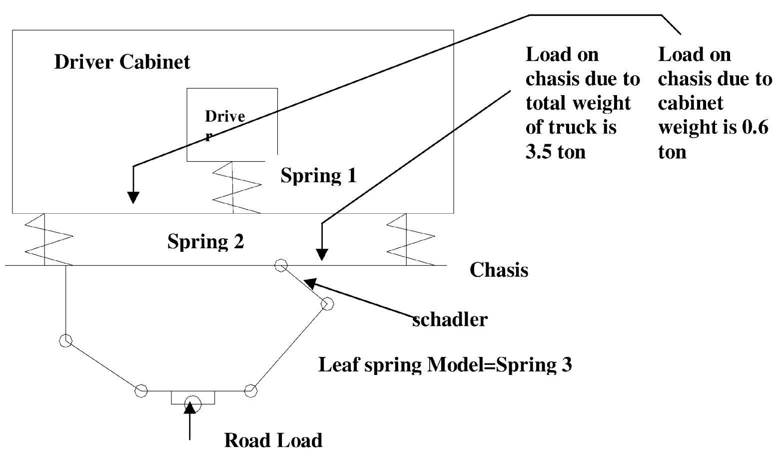


Table 3 Factors and their levels

#	*A* *Leaf spring length L* *(mm)*	*B* *Material property* E/S_y	*C* *Stiffness of spring 2* *(kN/m)*
1	1500	207E9/250E6	20
2	1550	207E9/275E6	30
3	1600	207E9/300E6	40
4	1650	207E9/325E6	50

In the model, the load applied on the leaf spring for this type of truck was 3.5 ton. The load of the front cabinet was 0.6 ton. In Figure 7 the constrained points are given as springs 1–3.

Among the Taguchi orthogonal tables, the appropriate one for four parameters with three levels for each is $L_{16}(4)$ (Bagchi, 1993). The parameters are assumed to be independent from each other. The results of the orthogonal table selection were obtained using a Linux Cluster Super Computer IBM × series 330 with 576 Mb RAM. The orthogonal experiment plan is shown in Table 4.

5 Results of optimisation

The new model was run for the 16 different cases given in Table 4 and the results are shown in Table 5. The constraints $(a_{rms})_1$, $(a_{rms})_2$ and $(a_{rms})_3$ are the root mean square values of accelerations at spring 1, spring 2 and spring 3 respectively, for the time interval tested.

Table 4 Orthogonal plan for experiments

	Parameters		
Number	*A (mm)*	*B*	*C (kN/m)*
1	1	2	3
2	3	4	1
3	2	4	3
4	4	2	1
5	1	3	1
6	3	1	3
7	2	1	1
8	4	3	3
9	1	1	4
10	3	3	2
11	2	3	4
12	4	1	2
13	1	4	2
14	3	2	4
15	2	2	2
16	4	4	4

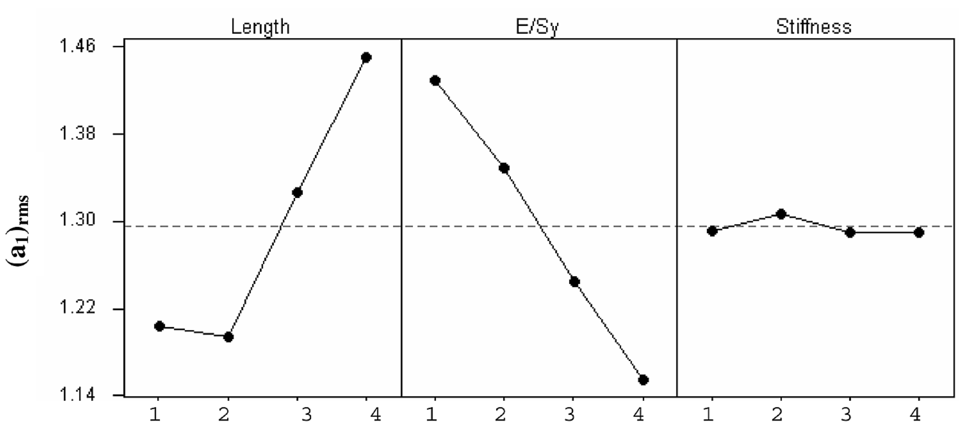
Table 5 Results of orthogonal plan

Number	Constraints		
	$(a_1)_{rms}$	$(a_2)_{rms}$	$(a_3)_{rms}$
1	1.29	0.80	0.347
2	1.28	0.40	0.272
3	1.19	0.35	0.211
4	1.06	0.15	0.136
5	1.38	1.12	0.287
6	1.25	1.10	0.226
7	1.16	0.45	0.121
8	0.99	0.70	0.121
9	1.49	1.15	0.226
10	1.38	0.70	0.167
11	1.25	0.80	0.186
12	1.19	0.70	0.075
13	1.56	0.90	0.211
14	1.49	1.12	0.181
15	1.38	1.15	0.196
16	1.38	1.09	0.257

5.1 General linear model: $(a_1)_{rms}$ versus length; E/Sy; stiffness

The effects of parameters on the first constraint are given in Figure 8. The higher the leaf spring length the higher is the change in acceleration in the chassis. On the contrary, increasing the strength of the leaf spring will help to reduce the acceleration changes. The effect of the stiffness of the spring under the front panel can be neglected, as expected.

Figure 8 LS-means for $(a_1)_{rms}$ (acceleration on chassis)



Statistical analyses were performed by using Minitab software (1998). The variance analyses made on the data predicts that at 99% the factors length and E/Sy are very significant whereas the stiffness is not significant.

The normal probability distribution of residuals and the histogram of residuals are also given in Figures 9 and 10 respectively to prove the accuracy of the distribution of the data obtained from model simulations.

Figure 9 Normal probability distribution of residuals for $(a_1)_{rms}$

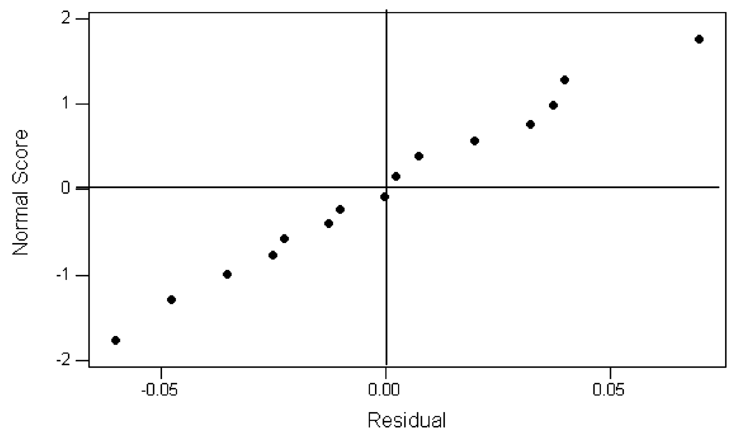
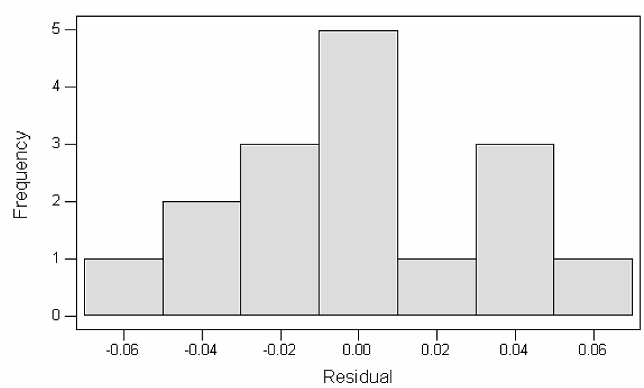


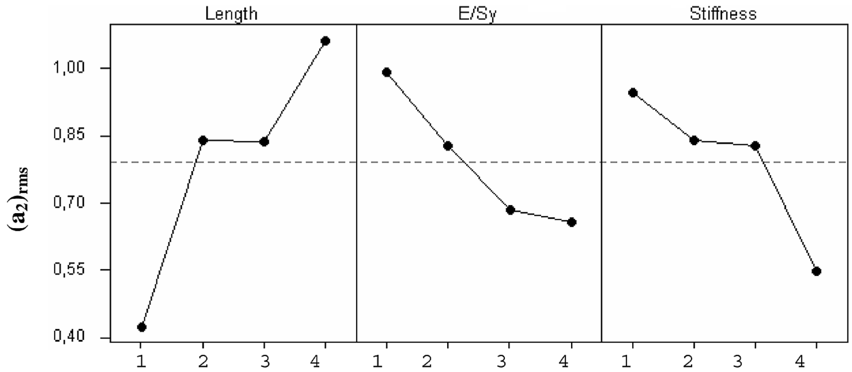
Figure 10 Histogram of residuals for $(a_1)_{rms}$



5.2 *General linear model: $(a_2)_{rms}$ versus length; E/Sy; stiffness*

The effects of parameters on the second constraint, $(a_2)_{rms}$ are given in Figure 11. The higher the leaf spring length, the higher the change in acceleration in the chassis again. On the contrary, increasing the strength of the leaf spring will help to reduce the acceleration changes. The effect of the stiffness of the spring under the front panel is also important. The higher the stiffness, the lower the acceleration changes. In order to predict the significance level of these factors for $(a_2)_{rms}$ the variance analysis was performed again. The calculations predict that at 90% length and stiffness are significant factors whereas E/Sy is not so important.

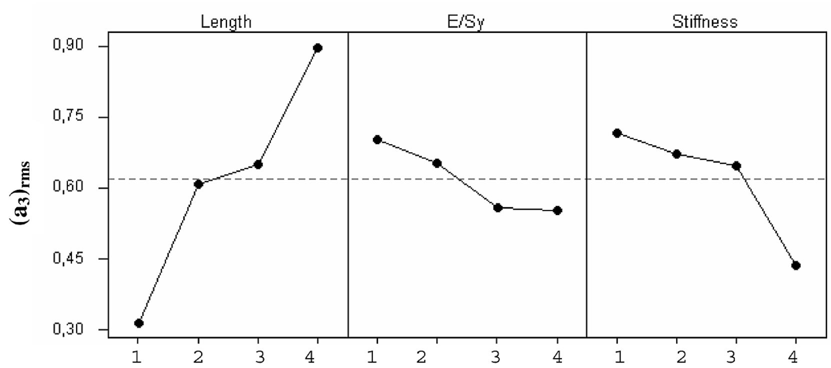
Figure 11 LS-means for $(a_2)_{rms}$ (acceleration on front panel)



5.3 General linear model: $(a_3)_{rms}$ versus length; E/Sy; stiffness

The above procedure was repeated again for the last constraint, $(a_3)_{rms}$. The effects of the factors studied on $(a_3)_{rms}$ are given in Figure 12. From the figure it is clear that the effects are similar to the ones for $(a_2)_{rms}$. However, the significance levels are very high with respect to $(a_2)_{rms}$.

Figure 12 LS-means for $(a_3)_{rms}$ (acceleration on driver seat)



The variance analyses show that even at 99% length and stiffness are very significant, at 95% E/Sy is also significant.

ANOVA calculations predict that $(a_1)_{rms}$ and $(a_2)_{rms}$ acceleration changes are mainly effected by the parameters studied. The acceleration under the driver's seat showed a behaviour dependent on the acceleration of the front panel. Therefore the correlation test was performed between $(a_2)_{rms}$ and $(a_3)_{rms}$ (Pearson correlation of $(a_2)_{rms}$ and $(a_3)_{rms} = 0.959$).

The Pearson calculation predicts a very strong correlation between $(a_2)_{rms}$ and $(a_3)_{rms}$. The correlation between a_{rms1} and a_{rms2} is 0.672. The value between $(a_1)_{rms}$ and $(a_3)_{rms}$ is 0.666. The correlations for these pairs are very weak. Therefore, that there is no correlation between them is assumed for the rest of the study. Multi-criteria analysis was performed for $(a_1)_{rms}$ and $(a_2)_{rms}$. The values obtained for the MANOVA calculation are assumed to be the same for $(a_2)_{rms}$ and $(a_3)_{rms}$.

5.4 Multi-criteria optimisation test

5.4.1 General linear model: a_{rms1}; a_{rms2} versus length; E/Sy; stiffness

The MANOVA test was performed for a_{rms1} and a_{rms2} as constraints. The parameters are again length, E/S_y and stiffness. The calculations are given in Tables 6–8 for length, E/S_y and Stiffness, respectively.

Table 6 MANOVA for length (s = 2, m = 0.0, n = 1.5)

Criterion	*Test statistic*	*F*	*DF*	*P*
Wilk's	0.01621	11.423	(6; 10)	0.001
Lawley-Hotelling	18.05066	12.034	(6; 8)	0.001
Pillai's	1.67494	10.306	(6; 12)	0.000
Roy's	15.25617			

Manova test parameter length is very significant at 99% for $(a_1)_{rms}$ and $(a_2)_{rms}$ constraints. All of the criterion predicted this significance. Very high significance level is observed for length.

Table 7 MANOVA for E/S_y (s = 2, m = 0.0, n = 1.5)

Criterion	*Test statistic*	*F*	*DF*	*P*
Wilk's	0.06566	4.838	(6; 10)	0.014
Lawley-Hotelling	11.77068	7.847	(6; 8)	0.005
Pillai's	1.09586	2.424	(6; 12)	0.090
Roy's	11.55784			

The significance level for E/S_y is given in Table 11 and it is seen that the level is slightly reduced with respect to length. However, the MANOVA test parameter E/S_y is very significant at 90% for $(a_1)_{rms}$ and $(a_2)_{rms}$ constraints. This result is also valid for all criterions.

Manova test parameter stiffness is not significant. The surface roughness regression test was used in order to find the optimum values for defined $(a_1)_{rms}$, $(a_2)_{rms}$ and $(a_3)_{rms}$ values.

Table 8 MANOVA for stiffness (s = 2, m = 0.0, n = 1.5)

Criterion	*Test statistic*	*F*	*DF*	*P*
Wilk's	0.19315	2.126	(6; 10)	0.140
Lawley-Hotelling	4.00444	2.670	(6; 8)	0.100
Pillai's	0.84025	1.449	(6; 12)	0.275
Roy's	3.96078			

5.5 Response surface regression test

Since the multi-criteria effects of factors are seen by the MANOVA test, it is necessary to find the optimum values for any desired factor levels. In order to do this, the action response surface regression method is employed. This method can predict a polynomial for each constraint. There are linear square terms in this polynomial consisting of the factors studied (length, E/S_y and stiffness).

The variance analyses of these terms and their significance are evaluated. In this method, if the polynomial terms are found to be significant they are used for any optimum value calculations.

To perform this test a new design for simulations is needed. The new design is given in Table 9. Since these are not experiments the runs are not randomised. The values of parameters are given as coded.

The code −1 is for the minimum level of the factor studied and 1 is for maximum value. For instance, for length, factor −1 means 500 mm and 1 means 650 mm. The values less than −1 and higher than 1 are advised from the method.

Twenty new simulations were performed. The results obtained from these simulations are also given in Table 9.

Table 9 New design of simulations for parameters and results obtained

Std order	*Run order*	*Blocks*	*Length*	E/S_y	*Stiffness*	$(a_1)_{rms}$	$(a_2)_{rms}$	$(a_3)_{rms}$
1	1	1	−1.00000	−1.00000	−1.00000	1.38	1.09	0.92
2	2	1	1.00000	−1.00000	−1.00000	1.06	0.25	0.33
3	3	1	−1.00000	1.00000	−1.00000	1.56	0.90	0.76
4	4	1	1.00000	1.00000	−1.00000	1.25	0.45	0.38
5	5	1	−1.00000	−1.00000	1.00000	1.38	0.86	0.78
6	6	1	1.00000	−1.00000	1.00000	1.06	0.15	0.12
7	7	1	−1.00000	1.00000	1.00000	1.30	1.40	0.70
8	8	1	1.00000	1.00000	1.00000	1.00	0.40	0.54
9	9	1	−1.68179	0.00000	0.00000	1.23	0.99	0.65
10	10	1	1.68179	0.00000	0.00000	1.25	1.12	0.75
11	11	1	0.00000	−1.68179	0.00000	1.25	1.16	0.68
12	12	1	0.00000	1.68179	0.00000	1.25	0.84	0.48
13	13	1	0.00000	0.00000	−1.68179	1.25	0.95	0.52
14	14	1	0.00000	0.00000	1.68179	1.25	0.95	0.52
15	15	1	0.00000	0.00000	0.00000	1.5	0.95	0.52
16	16	1	0.00000	0.00000	0.00000	1.25	0.95	0.52
17	17	1	0.00000	0.00000	0.00000	1.25	0.95	0.52
18	18	1	0.00000	0.00000	0.00000	1.25	0.95	0.52
19	19	1	0.00000	0.00000	0.00000	1.38	1.09	0.92
20	20	1	0.00000	0.00000	0.00000	1.06	0.25	0.33

5.5.1 *Response surface regression: $(a_1)_{rms}$ versus length; E/S_y; stiffness*

The results for new simulations are given in Table 9 and variance analyses is again employed. The analysis was done using coded units. The terms found for $(a_1)_{rms}$ are given in Table 10. There are linear, square and multiplication terms listed in the table. Variance analyses show the significance levels of the terms obtained for the polynomial of $(a_1)_{rms}$.

Table 10 Estimated regression coefficients for $(a_1)_{rms}$

Term	*Coef*	*SE Coef*	*T*	*P*
Constant	1.2466	0.03735	33.373	0.000
Length	−0.1270	0.02478	−5.125	0.000
E/Sy	−0.0539	0.02478	−2.176	0.055
Stiffness	−0.0022	0.02478	−0.089	0.931
Length * Length	−0.0132	0.02413	−0.546	0.597
E/Sy * E/Sy	0.0187	0.02413	0.773	0.457
Stiffness * Stiffness	0.0222	0.02413	0.920	0.379
Length * E/Sy	−0.0062	0.03238	−0.193	0.851
Length * Stiffness	−0.0038	0.03238	−0.116	0.910
E/Sy * Stiffness	0.0037	0.03238	0.116	0.910

Note: S = 0.09159, R-Sq = 76.7%, R-Sq(adj) = 55.7%

Square terms are very significant, even at 99%, whereas the linear terms are significant at 98%, so the polynomial can be used with confidence for calculations of $(a_1)_{rms}$.

5.5.2 *Response surface regression: $(a_2)_{rms}$ versus length; E/S_y; stiffness*

The next polynomial calculation was performed by $(a_2)_{rms}$. The analysis was done using coded units again. The terms found are given in Table 11. Variance analyses were performed.

Table 11 Estimated regression coefficients for $(a_2)_{rms}$

Term	*Coef*	*SE Coef*	*T*	*P*
Constant	0.9613	0.08445	11.383	0.000
Length	−0.3128	0.05603	−5.583	0.000
E/Sy	−0.0125	0.05603	−0.224	0.827
Stiffness	−0.0665	0.05603	−1.187	0.263
Length * Length	−0.0917	0.05454	−1.682	0.124
E/Sy * E/Sy	−0.0369	0.05454	−0.677	0.514
Stiffness * Stiffness	−0.0564	0.05454	−1.033	0.326
Length * E/Sy	−0.0638	0.07321	−0.871	0.404
Length * Stiffness	0.0338	0.07321	0.461	0.655
E/Sy * Stiffness	−0.0363	0.07321	−0.495	0.631

Note: S = 0.2071, R-Sq = 79.0%, R-Sq(adj) = 60.1%

From variance analyses it can be said that only linear terms are significant, with the highest significance at 99%.

5.5.3 *Response surface regression: a_{rms3} versus length; E/S_y; stiffness*

For the last constraint, stiffness, the calculations were repeated. The terms found are given in Table 12. The variance analyses of these terms are calculated again. The results are similar to the ones for $(a_2)_{rms}$. The significant terms are again linear terms at 99%.

Table 12 Estimated regression coefficients for $(a_3)_{rms}$

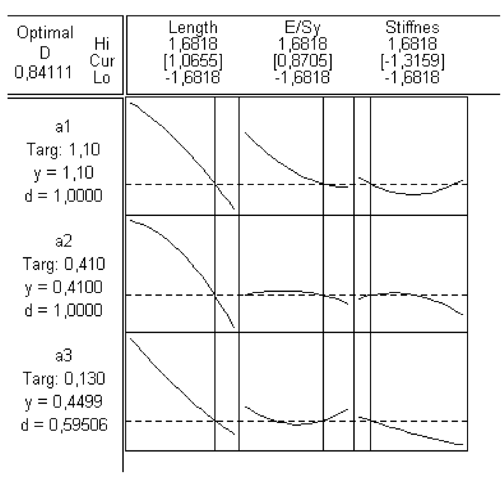
Term	*Coef*	*SE Coef*	*T*	*P*
Constant	0.5223	0.05890	8.868	0.000
Length	−0.1808	0.03908	−4.626	0.001
E/Sy	−0.0053	0.03908	−0.135	0.896
Stiffness	−0.0583	0.03908	−1.492	0.167
Length * Length	0.0200	0.03804	0.526	0.610
E/Sy * E/Sy	0.0483	0.03804	1.270	0.233
Stiffness * Stiffness	0.0059	0.03804	0.154	0.880
Length * E/Sy	−0.0375	0.05106	−0.734	0.480
Length * Stiffness	0.0150	0.05106	0.294	0.775
E/Sy * Stiffness	−0.0300	0.05106	−0.588	0.570

Note: S = 0.1444, R-Sq = 72.5%, R-Sq(adj) = 47.8%

5.6 *Surface optimiser*

At the end of these calculations optimum values are found for any constraint values by means of the surface optimiser method. In Figure 13 $(a_1)_{rms} = 1.1$, $(a_2)_{rms} = 0.41$ and $(a_3)_{rms} = 0.44$ are chosen. The corresponding factors length = 1.0655, E/S_y = 0.6705 and stiffness = −1.3159 are found as optimum values for this combination of constraints.

Figure 13 Surface optimiser method predicts the optimum values for any combination of constraints



6 Recommendations

The reason for this experimental and numerical study is to find the optimum values of the most important factors for the leaf-spring suspension system. Although the type of leaf-spring system, such as progressive, parabolic or trapezoidal is known as the main factor, it is not included in this study because of it is not possible to take into account in the three-link spring model. In order to perform this type of study, an explicit dynamic finite element method must be employed. The next research topic is to compare these leaf-spring systems in 3D by means of LS-DYNA 970 software.

7 Conclusion

The three-link leaf spring model is used to simulate the vehicle suspension system of a heavy commercial vehicle, based on the test data collected with most powerful and capable test bench. Real road-load data is collected with the LMS ROAD RUNNER data acquisition system. The model is made to run with ADAMS software. The verification of the model was provided by comparing the model results with road data. Then the model was used to optimise factors with three different constraints a_{rms1}, a_{rms2} and a_{rms3}. They are the root mean square values of the acceleration on the chassis, the front panel and the driver's seat respectively. The study points out that:

- The three-link leaf-spring model was good enough to be used in modelling purposes although the root mean square values slightly differ with the actual road values.
- The effects length, E/S_y and stiffness are investigated to find their affects on $(a_1)_{rms}$, $(a_2)_{rms}$ and $(a_3)_{rms}$. $(a_1)_{rms}$ is very sensitive to length and E/S_y but not to stiffness. The higher the length the higher $(a_1)_{rms}$ is but for E/S_y, the lower the E/S_y the higher is $(a_1)_{rms}$.
- a_{rms2} is sensitive to all of them. The higher the length, the higher the acceleration, but for E/Sy and stiffness the situation is reversed. The lower E/S_y and stiffness, the higher is the acceleration.
- $(a_3)_{rms}$ behaves similarly to $(a_2)_{rms}$. Therefore a correlation between them was suspected and a very strong correlation of 0.959 is found, whereas between the others ($(a_1)_{rms}$–$(a_3)_{rms}$ or $(a_1)_{rms}$–$(a_2)_{rms}$) there is no correlation.
- $(a_1)_{rms}$ and $(a_2)_{rms}$ are used for multi-criteria study by means of a MANOVA test. This test predicted that the length is very significant at 99% on $(a_1)_{rms}$ and $(a_2)_{rms}$, but E/S_y is significant at 90%. Stiffness shows no significance for $(a_1)_{rms}$ and $(a_2)_{rms}$.

- To find the optimum values for any combination of $(a_1)_{rms}$ and $(a_2)_{rms}$, the response surface regression test was necessary. For this reason a new experimental design was created and 20 new simulations were carried out. By means of these new runs, three polynomials for $(a_1)_{rms}$, $(a_2)_{rms}$ and $(a_3)_{rms}$ were obtained. The variance analyses of these polynomials show that the linear terms are significant. The regressions obtained are significant at 98 for $(a_1)_{rms}$, at 99 for $(a_2)_{rms}$ and at 95 for $(a_3)_{rms}$.
- Finally the surface optimiser method uses these polynomials to obtain the optimum surface. As an example, in Figure 13 $(a_1)_{rms} = 1.1$, $(a_2)_{rms} = 0.41$ and $(a_3)_{rms} = 0.44$ are chosen. The corresponding factors length = 1.0655, $E/S_y = 0.6705$ and stiffness = −1.3159, are found as optimum values for this combination of constraints.

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